

H1, H2 METHODS

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$$(H1) \quad \hat{G}(j\omega_n) = \frac{\sum y^{(k)}(n) \overline{u^{(k)}(n)}}{\sum |u^{(k)}(n)|^2}$$

$n \rightarrow \infty$

SAME PROPERTIES

$$(H2) \quad \hat{G}(j\omega_n) = \frac{\sum |y^{(k)}(n)|^2}{\sum u^{(k)}(n) \overline{y^{(k)}(n)}}$$

$$(H1) \quad \text{a.s. lim}_{M \rightarrow \infty} \hat{G}(j\omega_n) = \frac{\text{a.s. lim}_{M \rightarrow \infty} \frac{1}{M} \sum y^{(k)} \overline{u^{(k)}}}{\text{a.s. lim}_{M \rightarrow \infty} \frac{1}{M} \sum |u^{(k)}|^2}$$

$$= \frac{E\{y_0(n) \overline{u_0(n)}\} + \sigma_{y_0}^2(n)}{E\{|u_0(n)|^2\} + \sigma_u^2(n)}$$

AT THE RATE $\frac{1}{\sqrt{M}}$

$$= G_0(j\omega_n) \frac{1 + \sigma_{y_0}^2(n)/E\{y_0(n) \overline{u_0(n)}\}}{1 + \sigma_u^2(n)/E\{|u_0(n)|^2\}}$$

SYSTEMATIC ERROR

BIAS DUE TO $\frac{\sigma_{y_0}^2}{\sigma_{u_0}}$!

$$(H2) \quad \text{a.s. lim}_{M \rightarrow \infty} \hat{G}(j\omega_n) = G_0(j\omega_n) \frac{1 + \sigma_y^2(n)/E\{|y_0(n)|^2\}}{1 + \sigma_{y_0}^2(n)/E\{u_0(n) \overline{y_0(n)}\}}$$

IF $\sigma_{u_0}^2 = \phi$

$$H1: \frac{|G_0(j\omega_n)|}{|1 + \sigma_y^2/E\{|u_0|^2\}|}$$

\nearrow
1/SNR_{INPUT}

$$H2: |G_0(j\omega_n)| \left| 1 + \frac{\sigma_y^2}{E\{|y_0|^2\}} \right|$$

\nearrow
1/SNR_{OUTPUT}

$$|\text{a.s. lim}_{M \rightarrow \infty} \hat{G}_{H1}| \leq |G_0| \leq |\text{a.s. lim}_{M \rightarrow \infty} \hat{G}_{H2}|$$