

STANDARD LINEAR CLASSIFIER:

$$y = \underline{w}^T \underline{x}$$

(5)

PROJECTION TO MAXIMIZE THE CLASS SEPARATION (?)

MAXIMAL SEPARATION OF CLASS MEANS

$$\underline{m}_1 = \frac{1}{N_1} \sum_{\underline{x}_n \in C_1} \underline{x}_n$$

$$\underline{m}_2 = \frac{1}{N_2} \sum_{\underline{x}_n \in C_2} \underline{x}_n$$

$$\text{MAX}(\underline{m}_2 - \underline{m}_1) = \text{MAX}(\underline{w}^T(\underline{m}_2 - \underline{m}_1))$$

MEANS AFTER PROJECTION

$$\|\underline{w}\| = 1$$

$$\underline{w}^* \propto (\underline{m}_2 - \underline{m}_1)$$

STILL MUCH OF OVERLAP ERROR

FISHER'S DISCRIMINANT LINEAR

LARGE SEPARATION BETWEEN PROJECTED CLASS MEANS

SMALL IN-CLASS VARIANCE - MIN. OVERLAP

K=2

$$J(\underline{w}) = \frac{(\underline{m}_2 - \underline{m}_1)^2}{S_1^2 + S_2^2}$$

$$S_{1/2}^2 = \sum_{C_1/C_2} (y_n - m_k)^2 \quad y_n = \underline{w}^T \underline{x}_n$$

(RAYLEIGH QUOTIENT)

$$= \frac{\underline{w}^T \underline{S}_B \underline{w}}{\underline{w}^T \underline{S}_W \underline{w}}$$

BETWEEN CLASS COVARIANCE

$$= \text{MAX}$$

WITHIN CLASS COVARIANCE

$$\underline{S}_B \underline{w} \sim \underline{m}_2 - \underline{m}_1$$

$\|\underline{w}\|$ NOT IMPORTANT



$$\underline{w} \propto \underline{S}_W^{-1} (\underline{m}_2 - \underline{m}_1)$$

(CH. 4.1.6)

K > 2

D > K

$$y_k = \underline{w}_k^T \underline{x}$$

$$k = 1, \dots, D' > 1$$

$$\underline{y} = \underline{W}^T \underline{x}$$

$$\underline{S}_W = \sum_{k=1}^K \underline{S}_k = \sum_{C_k} (\underline{x}_n - \underline{m}_k)(\underline{x}_n - \underline{m}_k)^T$$

$$\underline{S}_T = \underline{S}_W + \underline{S}_B = \sum_{k=1}^K (\underline{x}_n - \underline{m})(\underline{x}_n - \underline{m})^T$$

$$\left(\sum_{k=1}^K N_k (\underline{m}_k - \underline{m})(\underline{m}_k - \underline{m})^T \right)$$

$$J(\underline{w}) = \text{Tr} \{ (\underline{w} \underline{S}_W \underline{w}^T)^{-1} (\underline{w} \underline{S}_B \underline{w}^T) \}$$

$$\underline{w} = D' \text{ largest eigen vectors (for largest eigen values) of } \underline{S}_W^{-1} \underline{S}_B$$